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ABSTRACT

The University of Maryland offers a physics course as part of the Maryland Collaborative for Teachers' Preparation (MCTP) project. One of the course aims is to promote the learning of the concept of a function through the learning of physics. Students learn in small groups through problem solving and with the aid of microcomputer-based laboratories. Students are asked to examine and find connections between experiments, stories, graphs, and algebraic representations. Analysis of observations of students' group work in the course reveals that students' understanding of experiments and stories lead to different translations to graphical and algebraic representations. (Author/WRM)

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ED 446 939

The Role of Representations in Learning an Interdisciplinary Mathematics and Physics University Course

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The University of Maryland offers a physics course as part of the Maryland collaborative for teachers' preparation [MCTP] project. One of the course aims is to promote the learning of the concept of a function through the learning of physics. The students learn in small groups, through problem solving and with the aid of microcomputer based laboratories. Students are asked to examine and find connections between experiments, stories, graphs and algebraic representations. Analysis of observations of students' group work in the course revealed that experiments differed from stories in many characteristics as graphs differed from algebraic expressions. Similarities were found among the translation process from an experiment or a graph to a story or an algebraic representation. Similarities were also found between the opposing direction translations. These translations differed in many characteristics from translations between graphs to experiments and from translations between algebraic equations to stories. According to the above analysis the four situations -- experiment, story, graphical and algebraic representations -- can be presented has vertexes of a parallelogram. Each edge and diagonal of the parallelogram represents a bi-directional arrow of possible translations. Between parallel edges there are many similarities.

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Information on other research conducted in the MCTP can be obtained in the MCTP Web site: http://www.wam.umd.edu/~toh/MCTP.html.

The current reforms in mathematics and in science education call for making connections between mathematics and science, applying technology, and advocate learning by inquiry (National Council of Teachers of Mathematics [NCTM], 1989; American Association for the advancement of Science [AAAS], 1993; National Research Council [NRC], 1996). In the spirit of these reforms, the NSF funded project, "Maryland collaborative for teachers' preparation" [MCTP], prepares pre-service teachers to teach science and mathematics in grades 3-9. In their teaching they are expected to connect between the disciplines, and use technology. One goal is to have preservice teachers employing instructional strategies that are compatible with constructivism perspectives. For example, teaching through student centered problem solving in small groups, addressing conceptual change, and dealing with more fundamental principles and less memorization of facts.

The program of the MCTP project includes instruction that matches the goals. Believing that the preservice teachers may remodel their university instructors, special courses in science and mathematics were designed. Most of these courses were taught by faculty in an inquiry mode of instruction, utilizing an interdisciplinary approach and microcomputer based laboratory technology.

The research described below was set to examine the application of representations in an MCTP science course.

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METHOD

http://www.physics.umd.edu/rgroups/ripe/jwl/representations/SH%26L.htm

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A case-study approach was used in studying an MCTP introductory physics class, during the Spring of 1996. The course has been given since Fall 1995, in the department of physics at the university of Maryland.

The goals for the course were to model the inquiry approach by creating a totally laboratory based course, by selecting a limited number of concepts, having the students work in small groups, and experience authentic assessments. Students also were expected to design experiments, carry them out, and present their results with evidence to support their conclusions. The course aimed to utilize science and mathematics in a seamless fashion, with students encouraged to work with science situations, and mathematical representations and to be able to make transformations between these. One of the major mathematics topics that was experienced is the concept of a function, especially linear.

To honor the goals, the students worked in groups of 3. The course met for three two hours laboratory periods, with no lecture, each week. Students' groups were given responsibility to carry out their work successfully with modest help from the instructor. Many of the experiments involved the microcomputer as a laboratory instrument for monitoring motion, and temperature related experiments. An example of a learning activity is given in Appendix A.

There were 29 students (18 Females, 11 Males) in the Spring of 1996. Eleven of the students were elementary education majors. The students are from different years in their undergraduate program. All students had completed at most one university mathematics course, and none had taken a university physics course.

During all lessons of Spring 1996 semester, one group was audio-taped. One of the researchers sat next to the group and wrote directly full transcripts. Unclear parts of these transcript were updated using the audio-tapes.

The group consisted of three female students: Mary (a 23 years old Caucasian), Jane (a Japanese student of 24 years old) and Anne (a 19 years old African-American and Hispanic). Mary and Jane took this course in the last semester of their undergraduate program, and Anne was a Sophomore. They all claimed in the beginning of the semester that they dislike and do not understand mathematics.

Analyzing the transcripts, we have observed that the lessons utilized two types of mathematical representations (graphical and algebraic) and two types of scientific situations (a real event and a story). Segments of a students' response to a question were classified according to the mathematical representations and the scientific situations that were dealt within them. The assignments that were given to the students served also as data to the research. The assignments were also classified according to the mathematical representations and the scientific situations that were dealt within them.

SENSORIAL VERSUS SYMBOLIC REPRESENTATIONS

In the observed lessons, the students encountered two types of scientific situations: an experiment and a story, and two types of mathematical representations: a graph and an experiment. In this chapter we will discuss the differences between experiments to stories, provided with evidence. We will show that experiments differed from stories as graphs differed from algebraic representations. According to the differences, we classify experiments and graphs as sensorial versus stories and algebraic representations as symbolic.

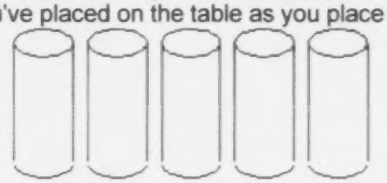
Experiments versus stories

Many papers in mathematics education that describe a process of building a mathematical model use one term to include both reality and realistic representation. For example, Lesh (1986) uses the term 'a given situation' and Blum and Niss (1991) use the term 'a real problem situation'. Janvier (1986) uses two terms 'situations' and 'verbal description', however he classifies them under the same category .

The different terms, used in the above papers, include both 'experiment' and a 'story' in them. We use the term 'experiment' in a limited context, as the actual physical event, that may include the process of carrying it out and collecting its results. The story is a description of a real or a an imaginary event. It summarizes the experiment with words and symbols.

We will give here two examples for stories and their matching experiments. The first example is drawn from the second lesson. In this lesson, students were provided with a description of a motion, "start 0.5 m from the detector, and walk from it with a slow steady pace." This description is a story that describes an experiment the students should try to carry out. The actual motion which the students performed is the experiment. Another example of a story is drawn from the sixth lesson. In question 1, Figure 1, the students were requested to write a story describing 9 stacks of washers. The actual stacking of washers, measurement of stacks, and collection of information is an experiment.

In this activity you'll place 2 cm stacks of washers on your lab table and examine the relationship between the number of stacks and the total number of washer on the table. Using a data table in your laboratory notebook, record the number of stacks and the total number of washer you've placed on the table as you place them on the table.



1 stack 2 stacks 3 stacks 4 stacks 5 stacks

1. Predict the number of washers you would need in order to make 9 stacks of washers. Then, carefully explain the thinking process that you used to make your prediction. List each step in the process and explain its purpose.

2. Write an equation you could use to predict the number of washers in 9 stacks.

1. Write an equation you could use to predict the number of washers in any number of stacks.

Fig. 1. First activity with equations -- stacking washers

The experiments conducted in the lessons differed from stories in several characteristics. These characteristics are listed below.

Sensorial versus symbolic

All the experiments that were conducted in the lesson were sensorial. One experiment dealt with temperatures, all other experiments were visual. The stories were predominantly verbal. The stories used key-words, such as time, distance, velocity and temperature, and symbols for their measurement units, such as sec, m, m/sec and °C.

Raw information

Experiments included all the detailed and raw information, were as stories were a simplification of the experiment. For example, when the student was walking her motion was not steady as requested in the story problem, since the student's walking is always variable. Another example is the story in question 1, Figure 1, and its matching experiment. The story describes stacks of a whole number of washers, with a 2 cm height. The matched experiment included the detail that a stack of a whole number of washers can not be 2 cm high. Students' were aware to this difference:

[Jane measured a stack of 2 cm as three washers.]

Jane: There is no way we can actually get 2 cm with these washers.

Mary: And this is 1.9. [height of stack.]

Anne: Excuse me?!

Jane: This one is 2 cm.

Mary: Not quite, it's almost 2 cm. It's like a millimeter off.

Anne: Yea.

Jane: It should be, because we had big washers, not like other groups.

Mary: Oh, okay.

Jane: It should be 3 washers for 2 cm stack.

Mary: Okay.

Physical boundaries

The experiments were limited by several boundaries. For example, the walking experiment was limited in the space available from the motion detector position to the wall of the class-room. It was also limited in the time available for answering each question in the lesson. In the group that was observed, Mary was walking much faster than Jane, so these limitations differed from student to student. Another example is that the experiment which the students conducted to match Figure 1 was limited by the amount of washers.

The story did not necessarily contain boundaries. One might assume that the motion is infinite, and that the number of washers is unlimited. Some of the stories stated boundaries explicitly, such as the starting position for the motion, or the number of desired stacks.

Numerical data

Most experiments involved also numerical data, such as time, distance, size, and temperature. Numerical data that was gained from experiments was based on measurements, while some stories included explicit numerical data.

We conclude that a story summarizes an experiment. Therefore, different stories may focus on and emphasize different characteristics of the experiment. That is, several different stories may summarize the same experiment. Since experiment includes detailed and raw information, several different experiments may be constructed to match to the same story.

Graphs versus algebraic representations

In the observed lessons the students encountered two types of mathematical representations: a graph and an algebraic representation. We use the term "graph" to describe a concrete drawing, and the term "algebraic representation" to describe a concrete formula or expression. Most of the graphs, that were included in the lessons, were used to describe an experiment or a story. These graphs differed from algebraic representations, as experiments differed from stories, as described below.

Sensorial versus symbolic

Graphs, like the experiments themselves, were visual, where as algebraic expressions, like stories used symbols. The students in the observations, used symbols and even words in algebraic expressions. They used symbols such as x, y, a and b to describe motion, and they wrote the equation below as an answer to

question 3 in Figure 1.

# of washers # of stacks = total # of washers

Raw information

Graphs, like experiments, included detailed and raw information were as algebraic representations, like stories, served for summary and simplification. Graphs, drawn by human or produced by the computer, were limited in their drawing accuracy. Therefore they included additional information.

Graphs that were produced by experiment, included even more raw information, that is collected from the original experiment. For example, the graph in Figure 2, like the experiment that produced it, includes the information that the student hesitated in the beginning, and slowed down by the end. It also indicates that the motion is not steady. A linear equation that represents the same graph, such as d=0.5+0.6t or d=0.5+vt, does not include all of this information.

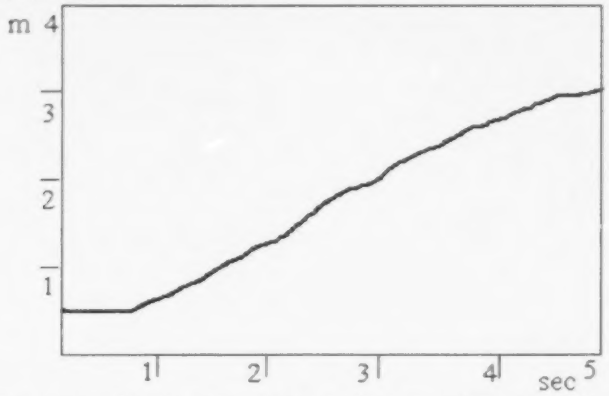


Fig. 2. A graph that the students obtained in the second lesson, from carrying an experiment that would match the story "start 0.5 m from the detector, and walk from it with a slow steady pace."

Physical boundaries

A concrete graph, in general, has a limited domain and range. Graphs obtained in the micro-computer based laboratory were further limited. They were limited to the motion detector range. Until the students handled technically the program the graphs were restricted to the given domain and range (for example, the graph in Figure 2, was set with a domain of 5 seconds, and a range of 4 meters).

An algebraic representation may be defined for an unbounded domain. The algebraic representation, like a story, may include boundaries that are explicitly described.

Numerical data

Some of the algebraic expressions included explicit numerical data, for the coefficients, where as numerical data that was gained from graphs was based on finding coordinates of points.

We conclude that, as with the relationship between experiments and stories, many algebraic expressions may represent the same graph, and many graphs may represent the same algebraic expression. Schwarz and Dreyfus (1995) have already discussed this ambiguity in general. They claim that there is ambiguity between graphs and algebraic representations for several reasons: (1) graphs are partial they are limited in domain, (2) there is a limit to the accuracy of the graph drawing, (3) frequently the domain of an algebraic representation is not specified, (4) there is no unique formula representing a function. We would like to add that there is more ambiguity between graphs and algebraic representations that represent the same physical situation, because of the technological boundaries in obtaining the graph, and due to the fact that the graphs include much more detailed information, as described in this chapter.

Summary

In this chapter we have presented characteristics of experiments, stories, graphs and algebraic expressions that were observed in the lessons. We described differences between experiments to stories, and between graphs to algebraic expressions. From the differences, similarities emerges between experiments to graphs and between stories to algebraic representations (see Table 1). Therefore, we use a single term "*sensorial* " to describe both experiments and graphs, and we use another term "*symbolic* " to describe both stories and algebraic representations.

Table 1

Observed characteristics of sensorial versus characteristics of symbolic representations.

Characteristics of sensorial representations (experiments and graphs)	*Characteristics of symbolic representations (stories and algebraic representations)*
Sensorial, visual.	Verbal, symbolic.
Includes detailed raw information such as accuracy of drawing, moving ability, size, etc.	Idealization and summary of information.
Limited by physical boundaries such as space, time, domain limit, etc.	May be unlimited. Boundaries may be specified explicitly.
Numerical data can be retrieved from it.	May include explicit numerical data.

TRANSLATIONS BETWEEN A GRAPH AND AN EXPERIMENT OR BETWEEN AN ALGEBRAIC REPRESENTATION AND A STORY

Translations between a graph and an experiment

In the previous chapter we have seen similarities between graphs and experiments. In the boundaries and in the details there is correlation between an experiment to its matching graph. Physical boundaries of time and space correspond to boundaries of domain and range respectively in a graph. Some of the physical detailed and raw information also appears in the graph.

In the lessons, translations from an experiments to a graph and vise versa were based on technical skills. Beginning with the first lesson, students' obtained graphs that were derived from experiments using the micro-computer based laboratory. Within this lesson, the students realized the function of the motion detector, as Jane first remarked, "I don't know, because it detects like the position were the object is, right, I don't know, I'm just guessing." In the second lesson, the students developed a technique for obtaining a graph, similar to one already shown on the computer screen, by walking in front of the motion detector. The student that was walking in front of the detector looked at the computer screen to see the new graph that was created due to her own motion. She walked faster or slower, closer or further, and stopped for more or less time according to the differences between the graphs. Using a trail and error process, the student obtained a graph that is close enough to the target graph. After several lessons, the student obtained graphs in just a few trials. In the third lesson, Mary tried to obtain the graph shown as the dark line in Figure 3. She walked backwards while looking at the screen, forming the graph that is shown as a dotted line in Figure 3. Mary explained her error, "Too fast." Jane added, "You should stop and come back slower."

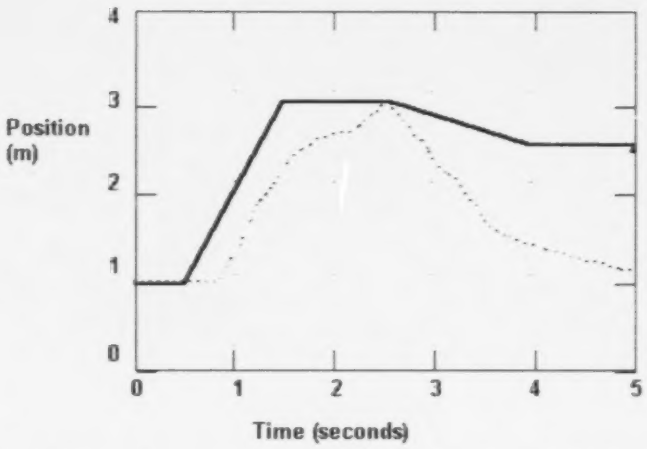


Fig. 3. Graph obtained (dotted line) in order to match the given graph (dark line), in the third lesson.

By the end of this lesson, Mary explained their method for carrying an experiment that will produce the desired graph:

TA: What do you need to do to make that curve?

Mary: You have the computer running, and then you try two or three times to get to it. But I don't know any other way.

TA: This is how you did it before?

Mary: Yes. But it was easy, because you had a point to stop.

In the beginning of the next lesson, Jane carried the same technique. She measured only where to start. The students repeated this technique in many more lessons.

[Anne opens a file, and the graph with dark line in Figure 4 appears on the screen.]

[Jane brings a meter stick, and leads it on the floor away from the motion detector.]

Mary: Were are we starting from?

Jane: 1m. [Jane stands for few seconds and than begins to walk away from the detector. Figure 4 a appears on the screen.]

Mary: Try faster.

Jane: [Jane walks and the resulting screen is as Figure 4 b.] I'm starting too slow.

Mary: And then you stop.

[Jane walks and forms doted line and the resulting screen is as Figure 4 c.]

Mary: Yea, now you got it.

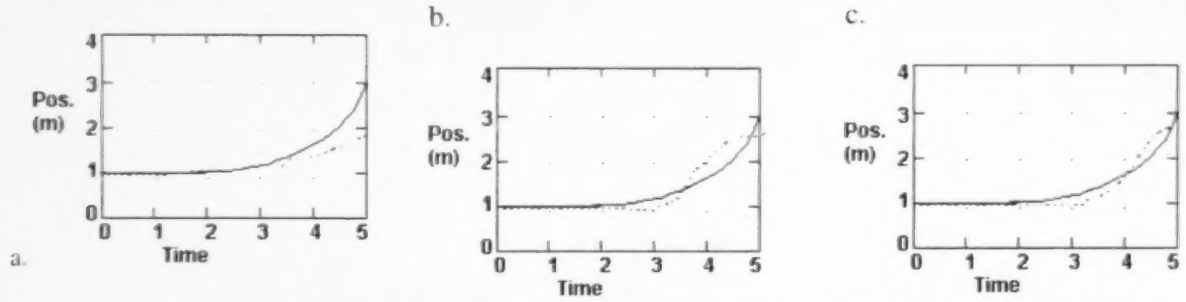


Fig. 4. Graph obtained (dotted line) in order to match the given graph (dark line), in the fourth lesson.

We conclude that the students had no difficulties during the lessons in translating between experiments and graphs. They used the computer to construct a graph that would match an experiment, and they developed a technique to construct experiment that will match a graph that appears on the computer screen.

Translations between an algebraic representation and a story

Algebraic representations and stories are symbolic. The students wrote equations using symbols, such as x, y, a, and b. In stories the students used key words such as time, distance, constant pace, and starting position. The students translated symbols to key words. For example, in the beginning of the eighth lesson, Mary wrote on the group's white board a summary of previous lesson,

y = ax +b, y = -.5x+3,

y = Position,

x= Time,

a = Constant pace or slope y/x,

b = Starting position or y intersect."

Later in this lesson, when asked "What is the physical meaning of slope?" Mary answered, "Slope, what does it mean now? It is constant pace." Jane confirmed, "I never thought what the slope is. Now it is," and she pointed on the group's white board. Anne also wrote the same answer in her notebook.

Preference of translations within the same type of representation

So far we have showed that translations within sensorial representations were based mainly on technical skills, and that translations within symbolic representations also involved a technique. In the examples below students were asked to translate between sensorial and symbolic representations. The students chose to translate within the same type of representation.

Preferring to translate between experiment and graph

When students were asked in the second lesson to predict graphs that will match stories, they actually imagined an experiment, as evident below.

[Students read the question, where they are asked to draw the two patterns they predict that will be obtained by the two motions: (a) Start 0.5 m from the detector, and walk away from it with a slow steady pace, and (b) Start 1.5 m from the detector, and walk away from it with the same slow steady pace.]

Anne: You walk slowly.

[Mary draws Figure 5 a, in her notebook.]

[Jane draws like Figure 5 a, with dashed lines, in her notebook.]

[Anne draws Figure 5 b, in her notebook.]

Mary: Here you will get first and it will be flat.

Anne: I understand.

Mary: It's true.

Anne: You mean you will get their before.

Mary: Were going to test it.

[Students wait till other group clears their way.]

[Anne writes, "If you start at 1.5 then it will take shorter distance and less time than if you start at 0.5." Anne changes the lower graph to a linear function starting at 0.5 as in Figure 5 c]

Teacher: Can I see your prediction?

Mary: I don't know why did I did it, but it should be straight. [Mary changes the upper graph to Figure 5 d.]

Jane: The second goes like that. [Jane copies Mary drawing to her notebook.]

Mary: But we didn't have space.

[Anne changes her drawing to look like Figure 5 d.]

[Teacher goes.]

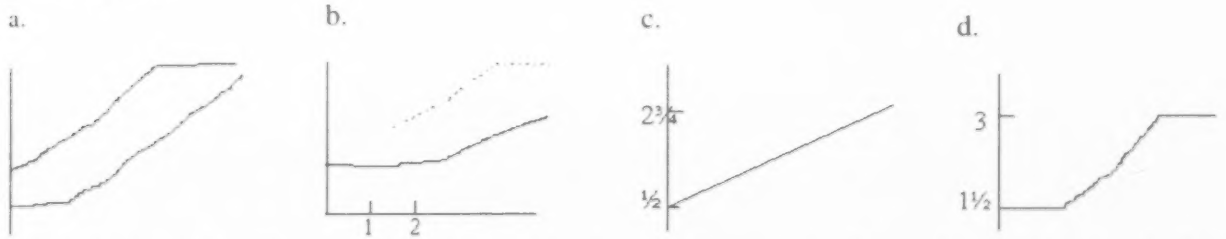


Fig. 5. Graphs predicted for two stories: (a) "Start 0.5 m from the detector, and walk away from it with a slow steady pace," and (b) "Start 1.5 m from the detector, and walk away from it with the same slow steady pace," in the second lesson.

[Students read the next question, where they are asked to draw the two patterns they predict that will be obtained by the two motions: (a) Start 0.5 m from the detector, and walk away from it with a slow steady pace, and (b) Start 0.5 m from the detector, and walk away from it with a medium fast steady pace.]

[Anne draws Figure 6 a, in her notebook]

[Mary draws Figure 6 b, in her notebook]

Jane: Didn't we do that?

Anne: The first one is the same.

Mary: The second is faster so you are going to hit first.

[Mary changes the upper pattern, and obtains Figure 6 c]

Teacher: How did you got this shape?

Anne: Move steady.

Jane: I hit the wall.

Fig. 6. Graphs predicted for two stories: (a) "Start 0.5 m from the detector, and walk away from it with a slow steady pace," and (b) "Start 0.5 m from the detector, and walk away from it with a medium fast steady pace," in the second lesson.

Preferring to translate between story and equation

The students chose to translate between a story to an equation instead of translating between a graph and an equation. In the beginning of the seventh lesson, the students were asked to "make a graph of motion at a constant pace" (see Appendix A). Then they were asked to analyze values of points on the graph (questions 1-4, Appendix A), check that there is a constant change (question 5) and even extrapolate values (question 6). Later they were asked to "Write an equation that describes the relationship between your position and the time" (question 7, Appendix A). The students then abandoned the graph and instead related to key words: distance, time and constant pace. That is they preferred to obtain the equation from a story over obtaining it from the graph. Doing so they recalled prior knowledge inadequately (Appendix B, lines 77-93). After the teacher have directed them to a more suitable equation, using the graph, they answered the rest of the question "Show that your equation is correct." Instead of checking the equation by translating it back to the graph they chose to translate the equation to a story that describes the motion they have carried (Appendix B, lines 192-207). By the end of the lesson, when asked the students insisted that they indeed checked the equation (Appendix B, lines 218-227, 245-254).

TRANSLATIONS BETWEEN SENSORIAL AND SYMBOLIC REPRESENTATION

Constructing symbolic representation to a given graph or experiment

When constructing an algebraic expression or a story, that would match a given a graph or an experiment, students summarized and simplified the graph or the experiment. They simplified details and raw information that were included in the graph or the experiment. They determined how and to what degree to simplify the given information, and how and to what degree to include details and elaborate the resulting representation. They also had to extrapolate beyond physical boundaries.

Simplifying original raw information

In order to obtain an algebraic expression or a story students changed their interpretation of reality and graphs, so the resulting representation will be simple.

For example, in the seventh lesson students walked toward the detector in a constant pace and obtained a graph as in Figure 12, Appendix B, on the computer screen. Analyzing the graph, they were asked to "Predict what your position would be after walking for 13 seconds?" (Appendix A, question 6). The students related to a different motion than they actually performed, since they would have been out of the detector's range after 13 seconds. They answered the question as if they have walked from the detector (Appendix B, lines 61-69). That is, in order to obtain a story that will describe the experiment they have carried, students changed their interpretation of reality.

The students were asked in the same lesson to "Choose several 1.0 s time intervals and calculate your change in position during each," and to obtain an equation that will describe the graph as in Figure 12 (Appendix A, questions 5 and 7). The students changed the information obtained from the graph, so it will be linear (Appendix B, lines 47-60,77-81, 94-103). Even the teacher offered the students to examine a linear graph in order to obtain an equation that matches a non-linear graph they obtained before from an experiment (see line 112 and the graphs in Figure 13, in Appendix B). That is, in order to obtain an equation that will describe the graph they had, students interpreted the graph as linear, although it wasn't.

Deciding whether to enrich the representation

When translating a graph to a story some information was not included in the story. The students had to determine whether to enrich the story so it will fit better the given graph. For example, in the fifth lesson the students were given a graph drawn on a paper. They were asked to describe the motion that would produce a similar graph. From the transcript of the fifth lesson that appears below, we can see that Jane was much more precise and gave more details than Mary.

Jane: So at 2 seconds it starts moving.

[Mary takes the paper from Jane and writes on it the distance 3]

Teacher [talks to all the class]: I don't want you to get into to much details but some thing you do. You will be graded on your directions, not on your succeed.

Mary: O.K. 4 after 2 seconds.

Jane: This looks like 3?.

Mary: What are we going to do?

Jane: Constant pace. You walk ? m in a 1? sec, in a constant pace.

Mary: We should mark this upon the floor, so we can know were to go.

[Mary writes step 1.5, step 4]

Jane: Do we need to explain what is a constant pace?

Anne: Just say which pace?

Mary: Slowly, for 1? sec.

Jane: Towards the mark. O.K. as soon as you hit there.

In the tenth lesson, Mary uses words such as 'gradually' instead of giving precise descriptions. Later, after the teacher commented on a different story, Mary chose a more detailed description.

In a similar manner when describing a curve, such as that in Figure 2, by an equation some information may be lost. The students need to choose between simplicity of the equation to better fitness to the graph.

(In the lessons we did not observed such hesitations, since the students mathematical knowledge was limited, and the lessons were restricted to algebraic representations by linear equations only.)

Extrapolating beyond physical boundaries

Graphs and experiments are limited by physical boundaries such as space, time, domain limit, etc. In order to find a matching algebraic expression or story that doesn't suffer from these limitations one should extrapolate the graph or the experiment. As an example of a need to extrapolate from an experiment see the following transcript from the sixth lesson. The students had to extrapolate how many washers are in 9 stacks when they did not have enough washers.

Jane: Two cm stacks means 3 washers.

Mary: So we need 27 of these?

Jane: Yes.

Anne: Three...

Jane: And we have just 11.

Anne: Yeah, but it doesn't say how many we already have.

Mary: This is nine stacks, so I guess we need 27.

Jane: Yeah, 27.

The students had also to extrapolate in order to carry the two motions: (a) "Start 0.5 m from the detector, and walk away from it with a slow steady pace", and (b) "Start 1.5 m from the detector, and walk away from it with the same slow steady pace." They extrapolated the obtained velocity-time graph that was obtained beyond the physical barriers that limited it. When Mary carried the experiment and obtained Figure 7, Anne explained "It went good, till you hit the wall," she explained the differences between the graph they obtained for the different stories, "Because we started further ahead and we had no room for that."

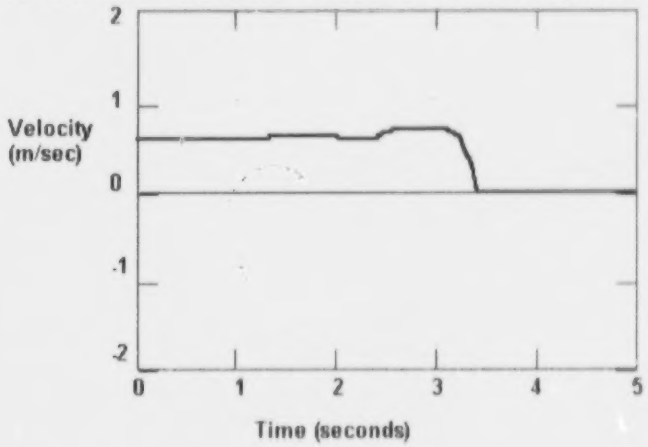


Fig. 7 . Velocity-time graph obtained for the story, "Start 1.5 m from the detector, and walk away from it with the same slow steady pace"

Another example of extrapolation from a graph we bring from the eighth lesson. The students created by an experiment the graph that is presented by a dark line in Figure 8. For this graph they have found an algebraic presentation as d=3-0.5t. This graph is presented by a doted line in Figure 8. When the students were asked to compare the graphs, they claimed that the new graph was obtained from extrapolating the

linear part of the original graph. They assumed that the starting position should have been 3 m away from the detector. Jane explained why they changed only the free-coefficient in order to obtain an equation that will match better the original graph, "Point 5 we found from the graph, but 3 we guessed."

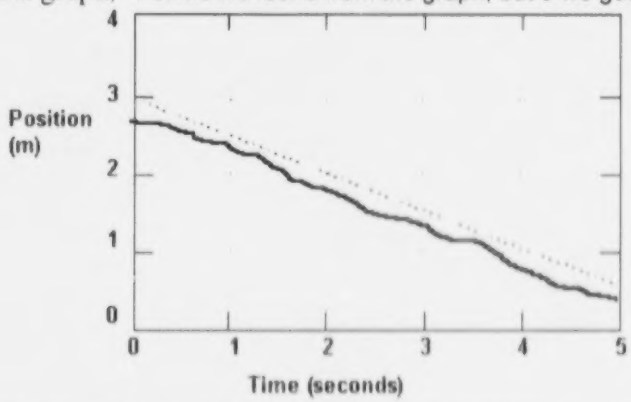


Fig. 8. Graph created by an experiment (dark line) and the graph of the equation d=3-0.5t (doted line)

Constructing graph or experiment that match a given symbolic representation

The process of constructing a graph or an experiment that would match a given algebraic expression or a given story was much less cognitive demanding than the other direction of translation. Experiments were constructed by following the description in the story. Graphs were constructed from equations, using the computer.

After translating the story into an experiment or a graph, the student had to determine whether the sensorial result in its boundaries and with the additional information matches the original story. The students most often decided to repeat the experiment and the graph construction, till it is sufficient. For example, in the seventh lesson the students were satisfied that the graph represents constant pace only in the fifth trial (Appendix B, lines 1-12). In the tenth lesson the students were asked to describe a motion that will result a graph as the dark line in Figure 9. Then they were asked to obtain the graph according to their description. Only the seventh trial was to their satisfaction:

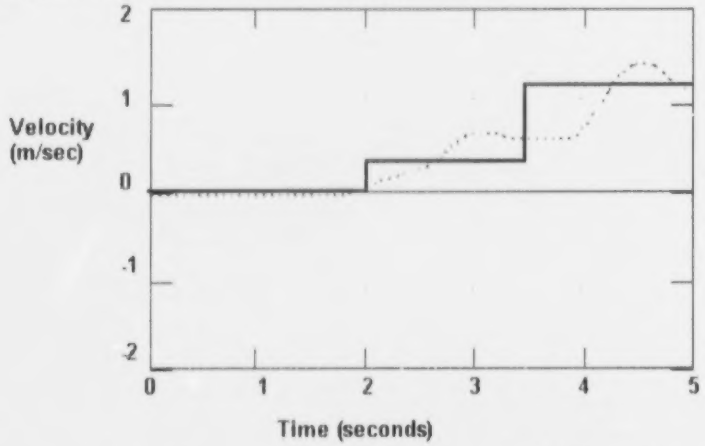


Fig. 9. Graph obtained (dotted line) in order to match the given graph (dark line), in the tenth lesson.

Jane: So, who's going to try this one?

Anne: I will. Okay, one minute. [Anne reads directions from her notebook.]

Anne: So basically I'm standing and then walk and then walk again.

Mary: No.

Anne: But it says stand still.

Mary: Then you...

[Anne walks. A graph with many noises is obtained.]

Jane: It was picking something else.

[Anne walks and obtains an almost linear graph.]

Jane: That was a little bit better.

[Anne walks and obtains a graph with many jumps on the last part.]

[Anne walks and obtains a graph almost linear.]

Jane: Can you see this? You were not looking.

Anne: I can, but it doesn't.

[Anne walks and obtains two curves, each reaching a step of the original graph.]

Jane: You run out of space.

Anne: You want to try?

[Jane walks and obtains a very low graph.]

Anne: Again.

[Jane walks and obtains Figure 9]

Anne: That's good.

Jane: Don't you want again?

Anne: That's basically the same shape.

In the fourth lesson, the students hesitated whether it is possible to create an experiment that will match the story. Whether it is possible to go back, stay there and return back to the detector, all within 5 seconds.

In the process of deciding whether the graph fits sufficiently the story, student become aware to causes of differences. For example, in the eleventh lesson Jane said, "We know that a wave is like a step, so it is okay," latter when constructing another graph she added, "I can't control the speed when I'm going so fast."

SUMMARY AND CONCLUSION

In this paper, we suggest to distinguish stories from experiments in science and mathematics education. We distinguish between story and experiment, since these two scientific situations have different characteristics, and they differ in the translation process between them to a mathematical representation.

As a case study, one reform-based physics course that employs mathematical modeling, was observed. In

the analysis of the observations we identified four situations -- experiment, story, graphical and algebraic representations. These situations are presented has vertexes of a parallelogram, in Figure 10. The upper situations in Figure 10 are considered as physical situations. The lower situations are mathematical representations.

In the lessons that were observed, the right situations in Figure 10 differed from the left situations. Experiments and graphs that were observed included detailed raw information, and they were bounded by physical constraints. Stories and algebraic representations serve as a summary of an idealized experiment or graph. They are unbounded, unless explicitly specified. Experiments and graphs are sensorial and visual, where as stories and algebraic representations are symbolic and verbal.

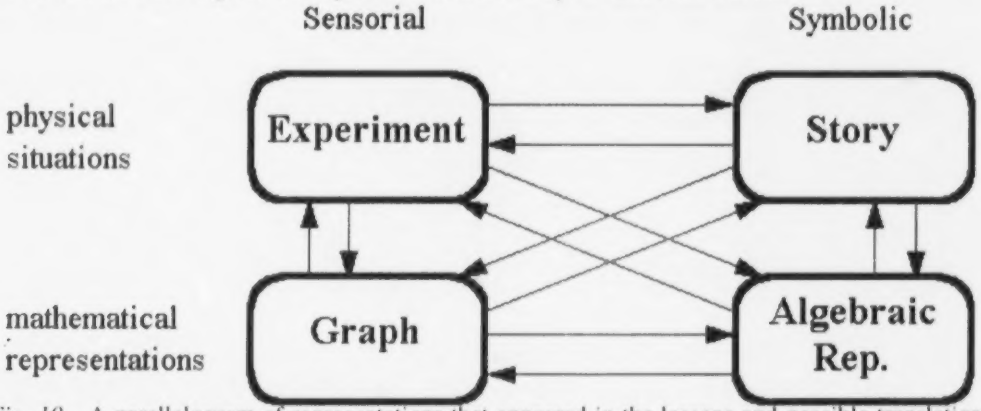


Fig. 10. A parallelogram of representations that appeared in the lessons and possible translations

Each directed arrow in Figure 10 represents a possible translations. Between parallel arrows, with similar direction, many similarities were found. Vertical arrows were much more easier to carry than the horizontal ones, or the diagonal. In the lessons, the translation between a graph and an experiment and the translation between an algebraic expression and a story were technical processes. The students avoided translation between an equation and a graph, and preferred to translate between the equation to a story. They also avoided translating from a story to a graph, and preferred to translate the story to an experiment and from it to a graph. The right to left translations, as in Figure 10, also could be carried technically. However in these translations the students had to determine whether the obtained result is sufficiently close to the original situation. The left to right translations, as in Figure 10, where the most cognitive demanding. When creating a symbolic representation that will match a sensorial one, the students had to simplify the given situation and to extrapolate from it. They also had to determine whether the obtained result is sufficiently close to the original situation.

From this we conclude that when combining mathematics education with science education one should identify between different scientific situations.

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APPENDIX A - SAMPLE PAGE OF ACTIVITY

Investigation 3: Equations

Activity 3 Walking at a Constant Pace

In this activity you'll create an equation that predicts the position of a person walking at a constant pace. Turn on your computer, run the *MacMotion* program, and make a graph of motion at a constant pace. When you're satisfied that you've walked correctly, save your graph and copy it into your laboratory notebook.

1. What was your position at 3.5 seconds? How did you determine this position from the graph?

2. What was the time when you reached a position 2.5 meters away from the detector? How did you find this time from the graph?

3. How long did it take for you to move between the two positions 1.0 m and 2.5 m in front of the detector? How did you find this time interval?

4. How far did you move during the time interval from 1.5 s to 3.5 s? How far do you think you would move during a 1.0 s interval? Why?

5. Choose several 1.0 s time intervals and calculate your change in position during each. Record the time intervals, your calculations and your position changes in a data table. How do your position changes compare?

6. Predict what your position would be after walking for 13 seconds? Then, carefully explain the thinking process that you used to make your prediction. Is it significant that you walked at a constant pace? Why or why not?

7. Write an equation that describes the relationship between your position and the time. Show that your equation is correct.

8. What units are associated with each number in your equation? What physical quantity is each associated with?

9. Obtain each group's equation for position and record it in your laboratory notebook. Then write one equation that could be used by any of the groups? Under what circumstances is this equation valid? Why?

APPENDIX B - SAMPLE TRANSCRIPT: GROUP WORK IN LESSON NO. 7

In this lesson, each group receives a computer with MacMotion program, connected to a motion detector, a 2*3 white board, eraser and dry markers. Each student had her calculator, notebook, and the page that appears in Appendix A.

The lesson started at 9.00_{AM} and ended at 10.50_{AM}.

1. Creating a position time graph of a constant pace walk.
2. [Students read silently the direction on the beginning of the activity.]
3. [Anne turns on computer and program.]
4. Mary: You are doing it so well.
5. [Jane walks and creates Figure 11a.]
6. Anne: You want to keep this one?
7. Jane: No.
8. Jane: [Jane walks and creates Figure 11b.] "Its not constant.
9. [Mary creates Figure 11c and then creates Figure 11d]
10. Mary: Tell me when it makes that noise.
11. [Anne says when to start, and Mary walks creating Figure 12 and saves it]
12. [The students are copying the graph by making a scatter plot for whole seconds, and then connecting the points]

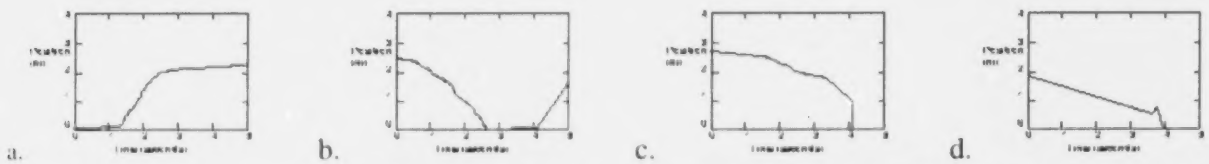

Fig. 11. Attempts to create a position time graph of a constant pace walk

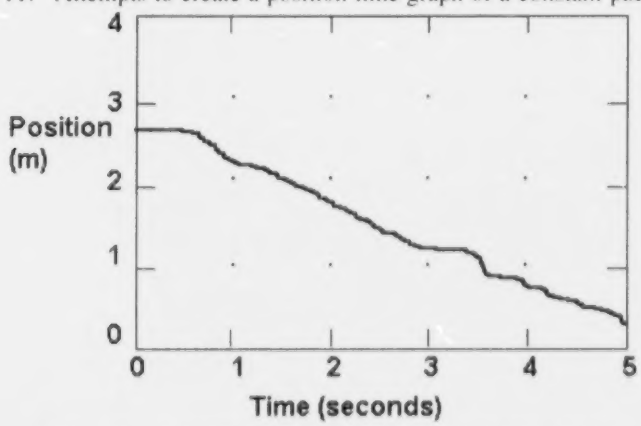

Fig. 12. A graph that represents a constant pace walk

1. Response to Appendix A, Q.1 -- position at 3.5 seconds
2. Mary: O.K. 'What was your position at 3.5 sec?' It is 1?
3. Anne: Yea, it says 1.
4. Mary: 'How did you determine this position from the graph?'
5. Jane: We looked at the line.

6. [Students write their answer to question 1 in their notebooks]

9.16

1. Response to Appendix A, Q.2 -- time when you reached a position 2.5 m
2. [Students answer separately question 2]
3. Anne: 2.5 m away is 1 sec?
4. Mary: Yea.
5. Teacher: ... I want one sentence response, with one sentence explanation. This shouldn't take you too long.
6. Jane: He said one sentence. I wrote only 2 sec.
7. Anne: It would hit about here. It is that much before 1. I would keep it 1.
8. Response to Appendix A, Q.3 - time from 1 m to 2.5 m.

9.21

1. Jane: How long did it take you to move from 1 m to 2.5 m?
2. [Students are writing answers in their note books. Mary writes the time at 1 m and 2.5 m]
3. Anne: What did you get?
4. [Students are looking at their notebooks for an answer]
5. Mary: For the 2.5m we said we have 1sec, and 1 m away what is your graph? What do you think is the time? 3.5 s?
6. Anne: Yea, 3.5.
7. Mary: So, just subtract the time.
8. [Students write their answer to question 3 in their notebooks]

9.25

1. [Mary finished writing] Mary: Are you doing number 4 yet?
2. Anne: I'm still doing number 3. [Jane and Anne are writing]
3. Anne: For number 3, what do we subtract, the mark between 1. to 2.5.
4. Mary: Not the meters, the seconds. In 1. you got 1 sec, and in 2.5 3.5 sec. So, it took you 2.5 sec to get from 1 to 2.5.
5. Response to Appendix A, Q. 4 - distance from 1.5 s to 3.5 s
6. Anne: I got 2.6. From 1.5. At 1.5 m I got ...
7. Mary: It is 1.5 sec, not m. So you get 2.25 m.
8. Jane: O, really?
9. Anne: What is it for 3.5?
10. Jane: 1 m.
11. Anne: 1.5 is 2.25. So, it is 2.25 minus 1, so it is 1.25.
12. Jane: O.K. 1.5 is 2 m, 3.5 is 1 m, so it is 1 m. It is 2 sec, so in 1 sec it is .5 m.
13. Response to Appendix A, Q. 5. - change in position in several 1 sec intervals

9.32

1. [Students are reading Q. 5 silently]
2. Anne: Are you writing your predictions on this graph, or doing a new one?
3. Jane: Do we have to do predictions? [Reading again]
4. [Jane draws a table, sec (1,2,3,4,5) versus distance (2.25, 1.75, 1.25, 0.75, 0.25), then she marks the differences in distances with > and calculates them] Jane: Wow! mine is really constant.
5. Anne: What are we supposed to do.
6. Jane: You should write the distances, and compute the differences. So mine should be a constant pace.
7. Anne: At 1 I got 2.5.
8. Jane: Yes, that's what you got in the beginning, but you can change this.
9. [Students are writing answers]
10. Anne: For 4 did we got 1 or bellow 1?
11. Mary: I wrote 1.

12. [Mary writes a table: seconds (1, 2, 3, 4, 5) versus distance (2.5, 2.0, 1.5, 1.0, .5)]
13. [Anne writes a table: seconds (1, 2, 3, 4, 5) versus distance (2.5, 2.0, 1.25, 1, .5), then she marks the differences (.5, .75, .25, .5)]
14. Response to Appendix A, Q. 6 - predicting position after walking for 13 seconds

9.40

1. Jane: Question number 6: After 13 sec we are going to hit the table.
2. [The students are glancing on other groups' screens, that shows graphs of a monotone growing functions.]
3. Jane: They had like that.
4. [Jane write 6.5]
5. Researcher: How did you get that?
6. Jane: We said that we are walking 0.5 m per sec, so after 13 sec we will have 6.5 m.
7. Researcher: You said you are going to hit the table.
8. Jane: I said I was going away from the detector, and starting at the detector.
9. (Simultaneous with the previous section) back to Q. 5.
10. Mary: Were are you?
11. Anne: Question 6.
12. Mary: What did you get?
13. Anne: I'm all lost... Between them I got .5, .75, .25.
14. Mary: So it is variegating, so write that.
15. Anne: I thought that in this activity you need to find the pace in order to predict your position.
16. Response to Appendix A, Q. 7 - An equation that relates between position and time.

9.51

1. Anne [to Jane]: Are you doing question number 7? There is no relation, no way to predict.
2. Jane: You can use my graph to answer question number 7.
3. Mary: You can write to number 7, 'If there was a constant pace.'
4. [Anne draws a new graph]
5. Jane: O.K. there is a one-on-one relationship, to get distance you multiply speed in time, that what we got here. [Points on her answer to Q. 6] "I learned in Japan that this is distance: to find distance you multiply speed and time. But it doesn't work here. [Points on the table in her answer to Q. 5. Jane draws on the white board a circle, and in its upper part "D", and "S and "T" bellow it.]

9.54

1. Mary: Yea, I heard that, but I think it is something else.
2. Jane: But it helps me a lot. [Jane erases the drawing she made on the white board]
3. [Mary writes on the white board: D = distance, T = time, C = constant pace, D = C * T]
4. Jane [to Anne]: Do you have any idea?
5. Anne: I like what she is doing.
6. [Mary writes on the white board: C = T/D, T = D/C] Mary: You can make three equations.
7. Jane: So our equation is correct. [Jane writes: D =distance, T =time, S =speed, D=S*T, T =D/S, S =D/T.] "It shows that it is correct.
8. Teacher: How things are going on?
9. Mary: We are on question 7. We got three equations.
10. Teacher: So you say there is no one way to write it?
11. Mary: Yes.
12. Teacher examines answers to Appendix A, Q. 6
13. Teacher: First, how do you got question number 6?
14. Mary: I said we had .5 m per sec, so we moved 6.5 sec.
15. Teacher: Does this make sense, Anne?
16. Anne: I did not get constant variation. [Anne reads her answer to Q. 5]
17. Teacher: How do you ...
18. Jane: Have different graphs.

19. Teacher: How can you compensate.
20. Jane: I looked at it as constant rate.
21. Teacher: Can it be that you make a little change rate.
22. Teacher helps students to answer Q. 7
23. Teacher: How is it similar to what we did with the plates?
24. Jane: The constant.
25. Teacher: Is it more like the one you had in activity 1 or 2?
26. Anne: The second.
27. Teacher: What was the difference between the two equations?
28. Anne: In the second one you added the incomplete file, but in the first not.
29. Teacher: Here also you had something in the beginning.
30. Jane: She didn't know when to start.
31. Teacher: Lets say it is straight in here. [Teacher. points on the graph's left edge.]
32. Mary: We would be starting at 3 m.
33. Teacher: So you had something.
34. Mary: What if we flipped the graph and started from zero up.
35. Teacher: Exactly at zero, or above?
36. Mary: Exactly.
37. Teacher: Then you would have what you got here. [Teacher. points on the equations] Think what will happen here. {Figure 12.} [Teacher. leaves the group]
38. Jane and Anne talk.

10.03

1. Mary: If you travel 1 m in 2 sec it is .5, and then you add it to 3?
2. [Jane writes on the white board: O*O+O=] Jane: He wants something times something, plus something, equals constant pace. Ho, it should be distance. [Jane erases "c" from the right circle and write "d" instead.] It is constant pace times time. [Jane writes: C*T+O = d] This is this, but what is here? [Jane draws Figure 13] Should I write here 3m or 1 sec?

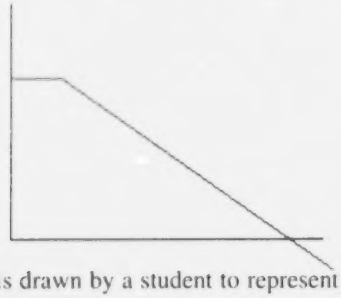

Fig. 13. A graph that is drawn by a student to represent the graph in Figure 12.

1. [Teacher. comes back] Teacher: How are you doing?
2. Jane: We don't know what to do with that. [Jane points on left edge of Figure 13 and then erases the board]
3. Teacher: Try to think on something like that, and then you could think on this. [Teacher. draws Figure 14a]
4. Teacher: If you have a graph like that, what is your equation? Write down. [Teacher. draws Figure 14b]
5. Jane: y=
6. Anne: x
7. [Jane writes y = x] Jane: Plus.
8. Anne: b
9. [Jane writes y = ax +b, then immediately erases the a.]
10. Teacher: Why did you erase that?
11. [Jane writes y = ax +b] Jane: It looks familiar.
12. Teacher: What are a and b here?

13. [Jane writes 1] Jane: a is the slope.
14. Teacher: What is the slope here?
15. Jane: You move 1 and up .5. [Jane writes .5x+1, then erases]
16. Teacher: Put it back.
17. [Jane writes .5x+1]
18. Teacher: How did you figure this out?
19. Anne: You substitute 0 in x place.
20. Teacher: Why did you do that?
21. Anne: That what you always do.
22. Jane: If x is zero then it is this point.
23. Teacher: Write this down.
24. [Anne writes y=1]
25. Teacher: What else can you do?
26. [Jane writes x=3, 0.5(2)+1=]
27. Teacher: What that gives you? What is the interpretation?
28. Jane: That's y, were x is 2.
29. Teacher: What would happen if you had a graph like this? [Teacher. draws Figure 14c] "It is more like yours.
30. Jane: The line goes down so it is a negative slope.
31. Teacher: What it is?
32. Mary: .5.
33. Jane: ? [Jane adds doted lines, as in Figure 14d]
34. Anne: I don't remember what is a slope.
35. Mary: Rise over run.
36. Teacher: Can you write this down.
37. Mary: I don't know. [Mary writes y2-y1 / x2-x1]
38. Teacher: We call it y over x.
39. [Mary writes y/ x]
40. Teacher: How do you do this here? What is y2-y1?
41. [Mary writes 3-2]
42. Teacher: What is x2-x1?
43. Mary: I don't know.
44. Teacher: Lets mark. [Teacher draws a new graph and marks points as in Figure 14e] How does this change?
45. Mary: It is [writes: 2-3 / 4-0 = -1/4]
46. Teacher: Does it make sense? How can you use it for equation.
47. Anne: ax +b [write y =ax +b]
48. Teacher: How do you use that?
49. Jane: That's slope
50. [Anne writes y=-?x]
51. Teacher: Now haw can you check if its all right?
52. Anne: At zero y=3.
53. Teacher: Can you try another point. By the way there can be many lines out there, but if have two points ...
54. [Anne writes y= -1/4(2)+3, -2/4+3]
55. Jane: Try to put 4 in.
56. Teacher: No, I won't let you go from it.
57. [Jane writes -1/2+3, y=2.5]
58. Teacher: I want you to do this, but instead of y over x, you have position and this is time. [Teacher. draws axis of Figure 14f] If it would go strait, were will it start?
59. Anne: 3.
60. [Teacher. marks the point (0,3) in Figure 14f] Teacher: And another point. At 5 what do you get?
61. Mary: Point 3.

10.23

1. Jane: Point 5.
2. Teacher: It is some were around here. [Teacher. completes the graph to Figure 14f.] Continue from here. [Teacher. leaves the group]
3. Anne I need to take math again.
4. Mary: I have forgot every thing.
5. Jane: The slope is negative. [Write is her notebook]
6. Researcher: Try it on the board first.
7. Jane: So you can see.
8. [Mary writes on the board y =ax +b] Mary: In our case [Mary writes: y =-ax +b, and then she writes y=-.5x+3]

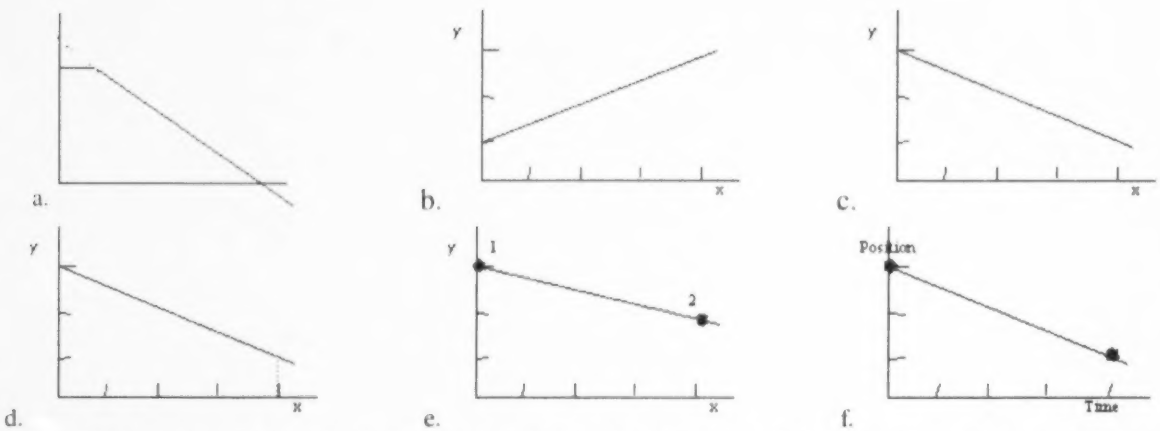

Fig. 14 . Graphs that were drawn by the teacher in order to help the students find the equation that matches the graph in Figure 12

1. Response to Appendix A, Second part of Q. 7 - Showing that the equation is correct
2. Mary: a is the rate in our situation. What is b? Distance?
3. Anne: Line intersection.
4. Mary: How can you explain that?
5. Anne: Position or some thing.
6. Jane: Starting position.
7. Mary: Is that fine? because if it went the other way [moving hand up] were b be the starting position?
8. Jane: Yes.
9. Mary: What is x?
10. Jane: x is the time. y is?
11. Mary: Position.
12. Jane: Do you write 3, if the line goes from 3?

10.28

1. Mary: Yea.
2. Researcher: Which question are you doing now?
3. Jane: 7, we need to show that the equation is correct.
4. [Students writes there explanation in the note books]
5. Response to Appendix A, Q. 8 - Associate units and physical quantities for equation
6. Mary: Do you want to do question 8?
7. Anne: Yea.
8. Mary: y is m. x is sec. so a is m per sec.
9. Anne: You said a is m/sec, y is m, and x is?
10. Mary: Seconds, because it cancels the seconds.
11. Jane: What physical quantity?
12. Anne: What units?
13. Jane: We just did this one.

14. Anne: So, physical quantity.
15. Response to Appendix A, Q. 7 - Showing that the equation is correct

10.35

1. Researcher: I think that in Q. 7 you are asked to show if the equation isn't wrong.
2. Anne: I think they don't need any right or wrong answers.
3. Mary: They didn't do so with our equation.
4. min. Small talk.]
5. Teacher. checks answer to Appendix A, Q. 7

10.36

1. [Teacher. comes back.] Teacher: You guise came up with an equation?
2. Mary: Yes, it is here. [points on the white board]
3. Teacher: Did you check it?
4. Mary: Yes.
5. Teacher: What are your variables here?
6. Anne: y, x, and a.
7. Teacher: Are those variables for this? [points on position versus time scale]
8. Jane: Yes.
9. Teacher: What if I write [changes graph variables to P and Teacher]
10. Anne: You can change it to P and Teacher.
11. Teacher: For what that is good?
12. Anne: It make sense from were it comes.
13. Teacher: What does -.5 represent?
14. Mary: Slope.
15. Jane: How fast you go. Per sec you were going .5 m.
16. Teacher: So, in a sec .5 m. What does the minus says?
17. Mary: We went away to the detector.
18. Teacher: How can you make it positive?
19. Mary: Walk from detector.

10.45

1. Researcher: Go back to your previous questions and check if they fit the equation.
2. [Students write answers to Q. 8 in their notebook]
3. [Students move to activity 4]
4. Researcher: Go back to Activity 3. When ever you change anything you should go back.
5. Jane: She just changed her table.
6. Researcher: No, the equation.
7. Jane: We really didn't change that.
8. Mary: Up to here we had just found that the constant pace is .5 and we didn't relate that to the equation.
9. Jane: That's O.K.
10. Jane: Are we supposed to do Activity 4.

10.48

1. Mary: We don't have time.